An introduction to time series and time series models

Applied Time Series Analysis for Ecologists

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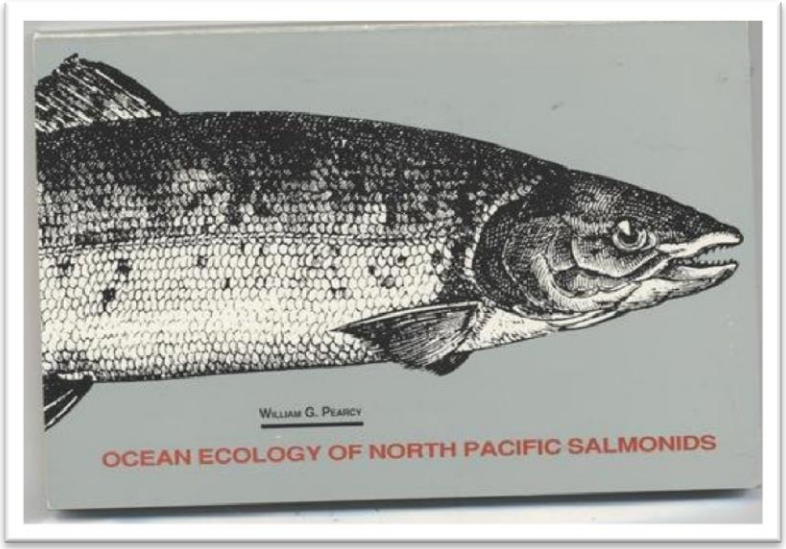
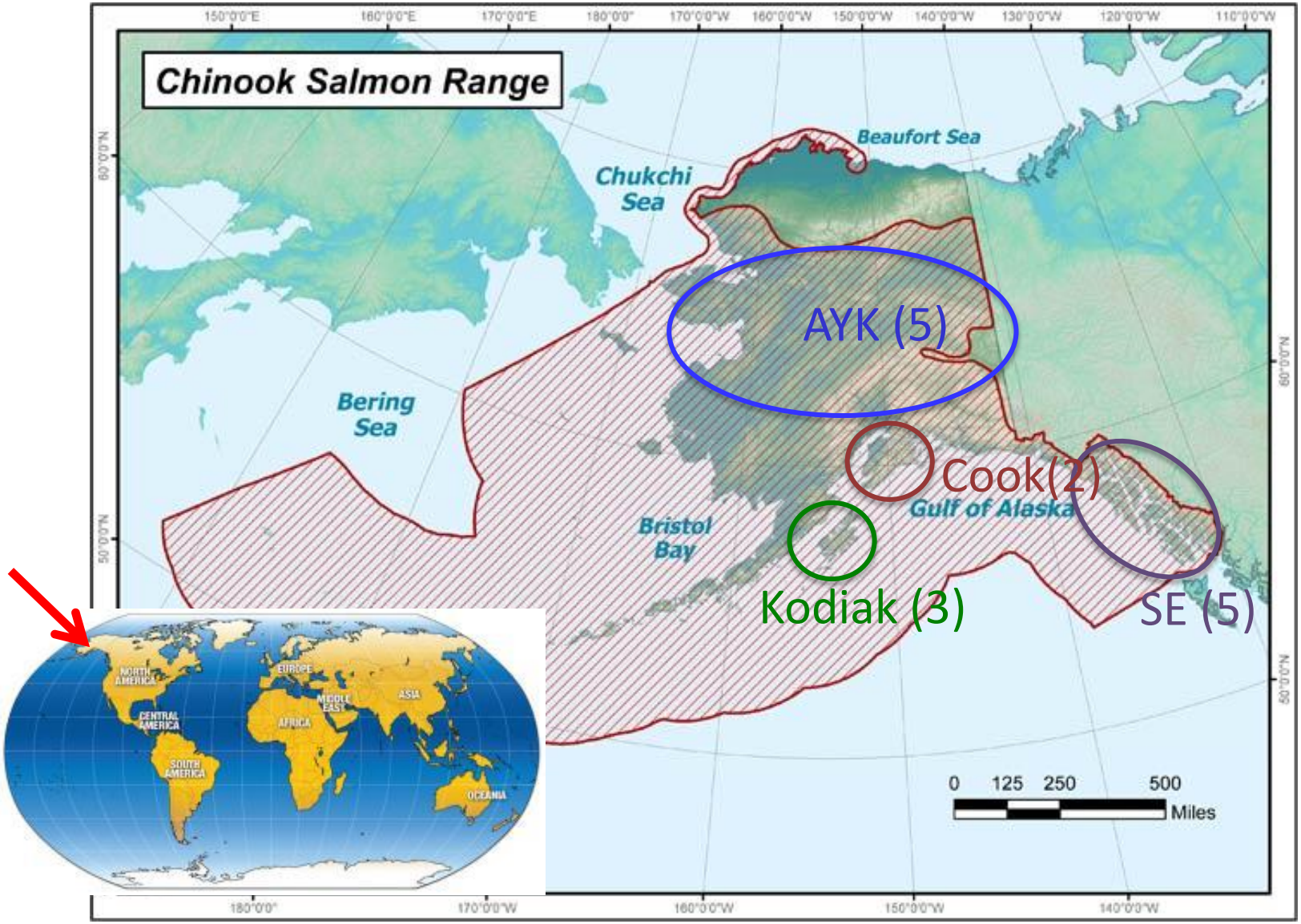
16-22 Sept 2015

An example of a study using time-series analysis work by Mark Scheuerell at NWFSC, Seattle

What evidence exists to support the hypothesis that large-scale ocean-climate drives fluctuations in Alaska salmon survival?

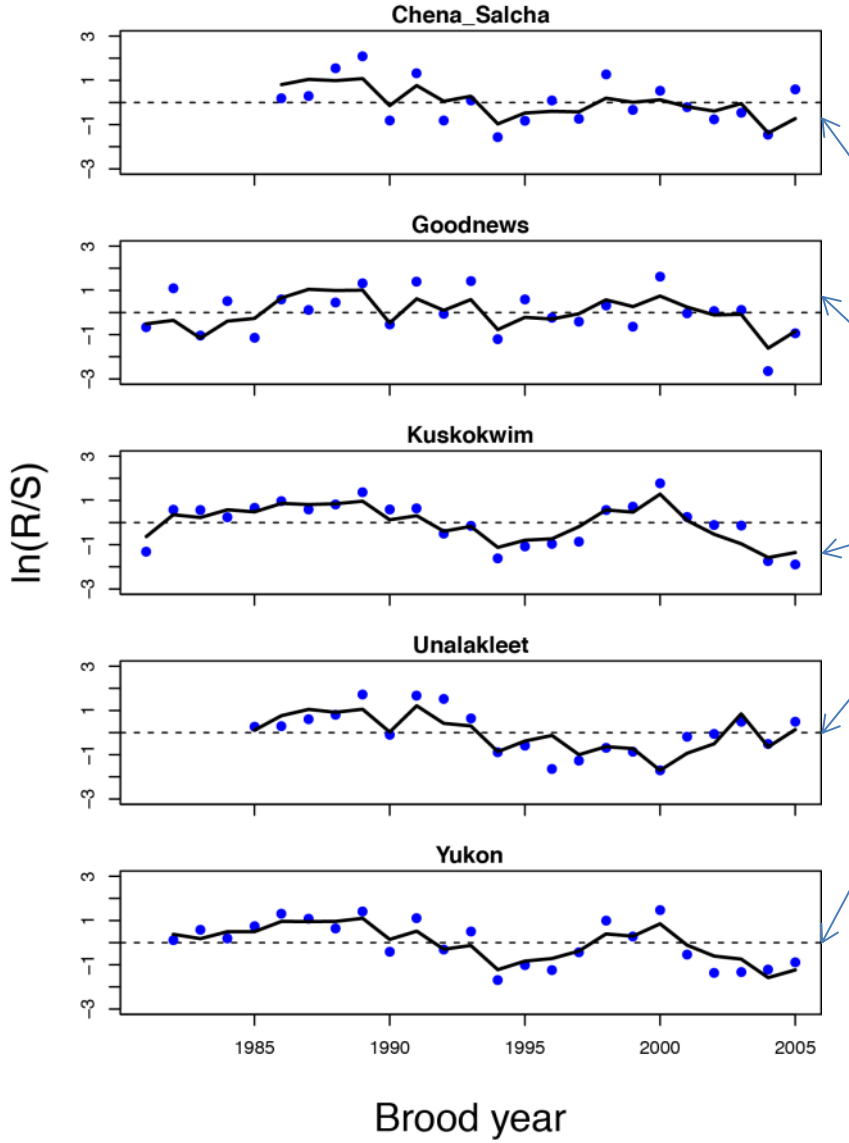


Alaska Chinook salmon



The data: time series of log recruits per spawner versus brood year for AYK region

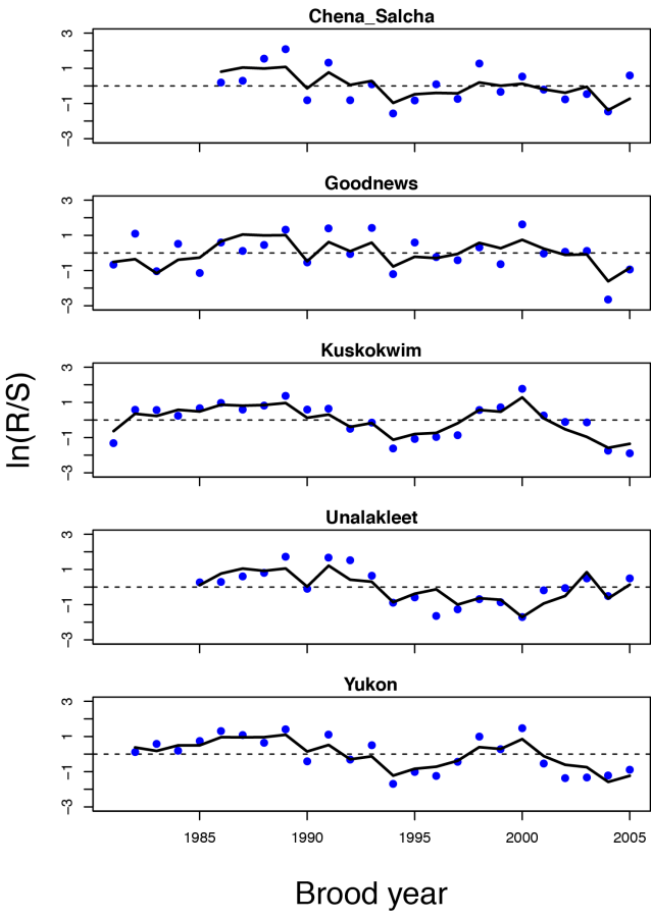
Estimate of survival



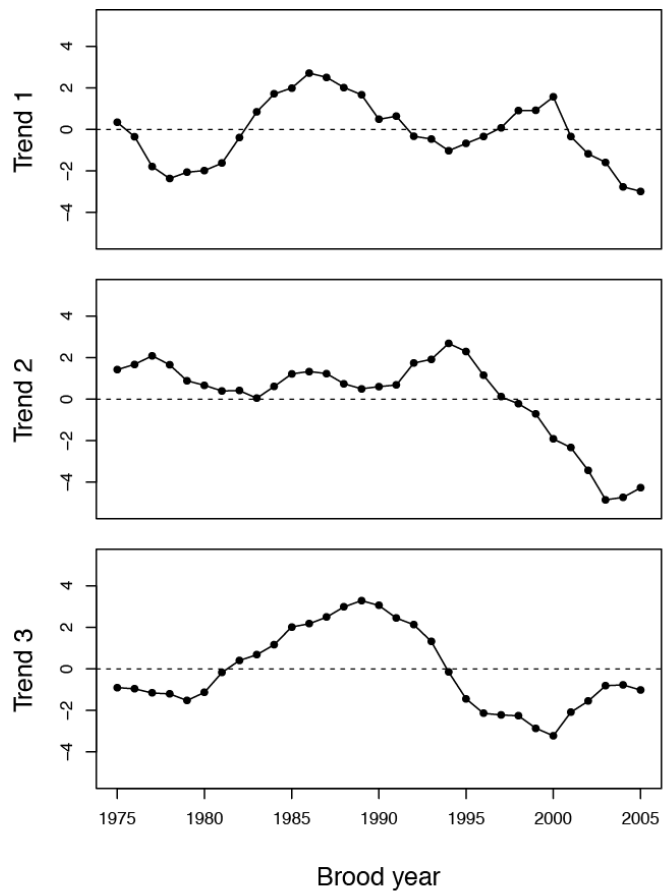
Different stocks/populations within one of the Alaska regions

The analysis
(Dynamic Factor Analysis)

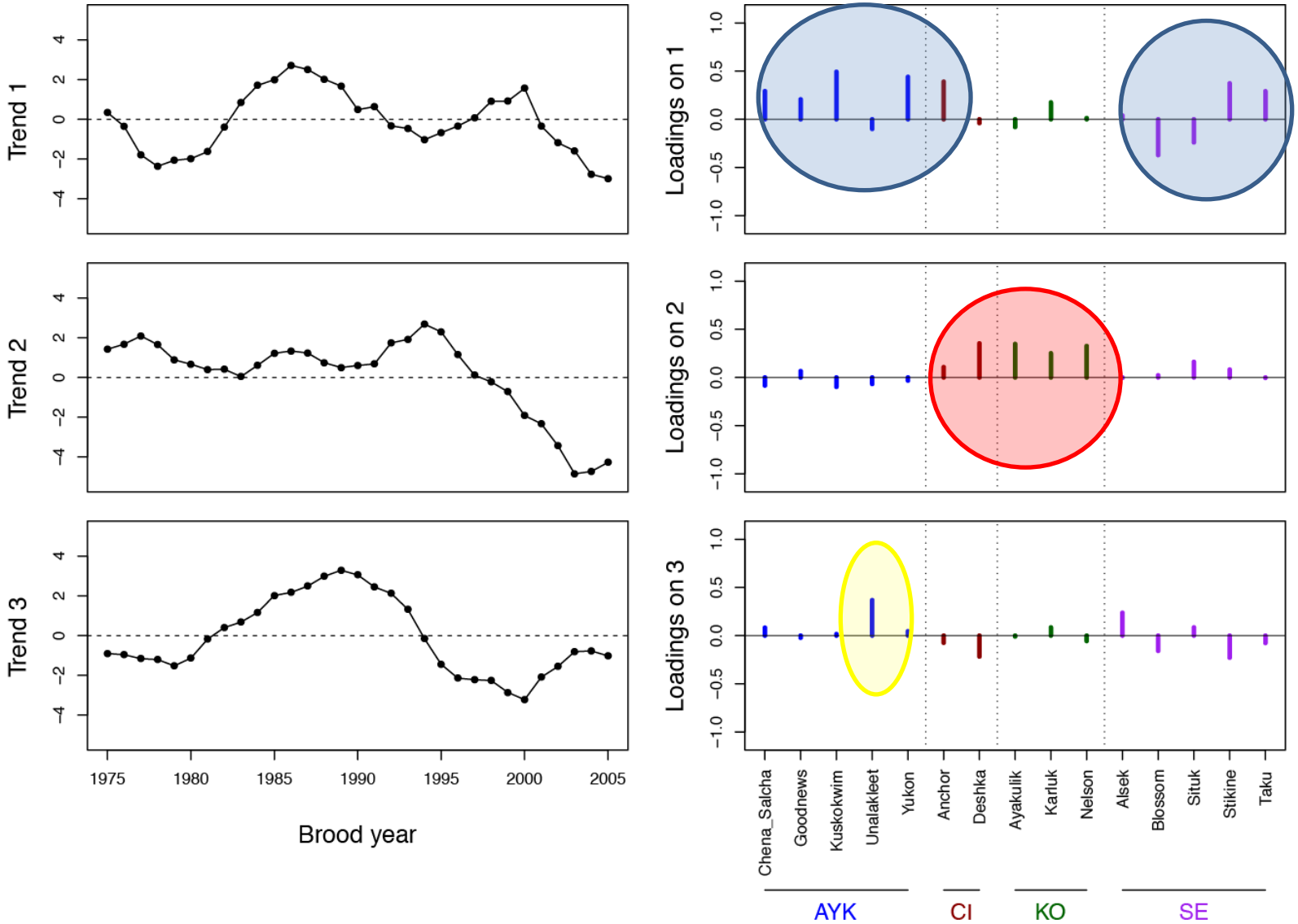
Raw Data: 15 time series
(5 of 15 shown)



Can be described by 3
overall patterns

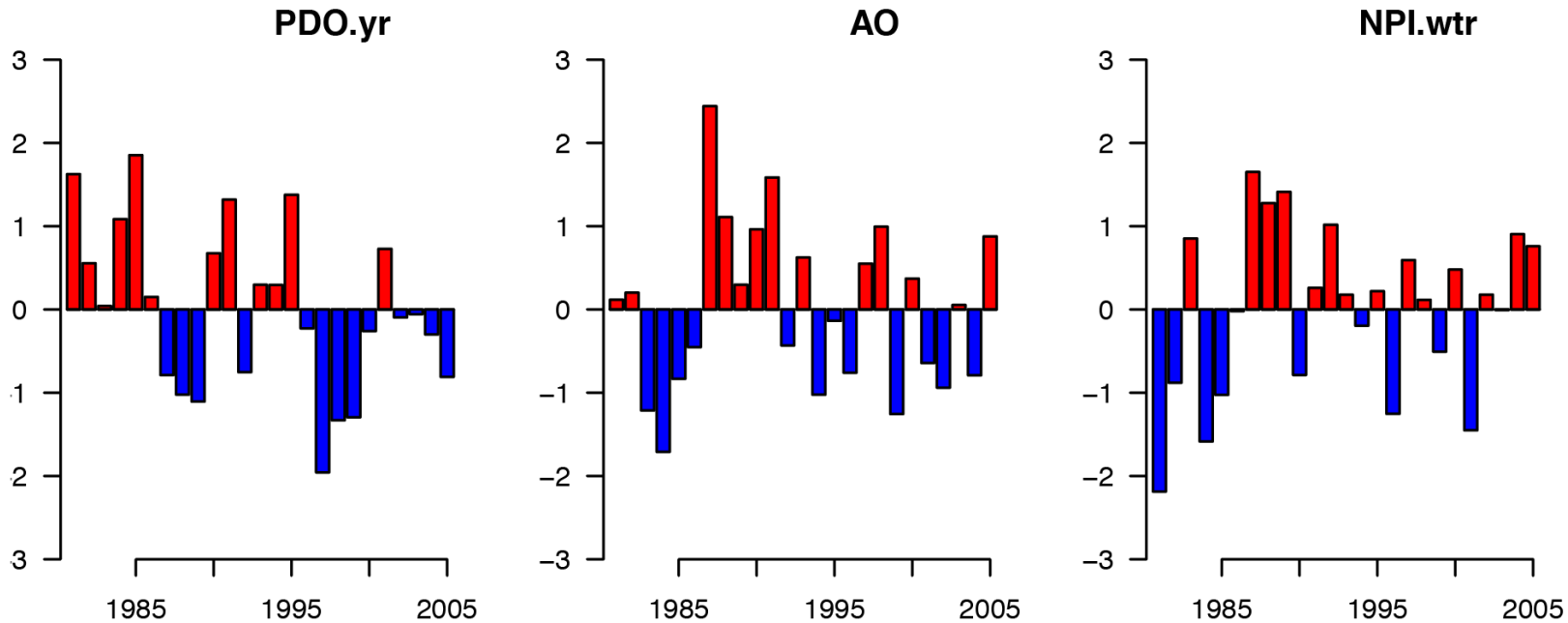


Results: 3 large-scale trends/drivers
The different stocks weight on these differently.



Next analysis looks at correlation between the overall trends and large-scale environmental indicators

- Pacific Decadal Oscillation
- Arctic Oscillation Index
- Aleutian Low Pressure
- North Pacific Index



Introduction to time-series analysis in R

- Characteristics of time series (ts)
  - What is a ts?
  - Classifying ts
  - Trends
  - Seasonality (periodicity)
  - Stationarity
- Time-series models
  - White noise
  - Random walks
  - Autoregressive (AR) models
  - Moving average (MA) models
  - ARMA models
- Diagnostics for time series
  - Autocorrelation functions (ACF)
  - Correlograms

What is a time series?

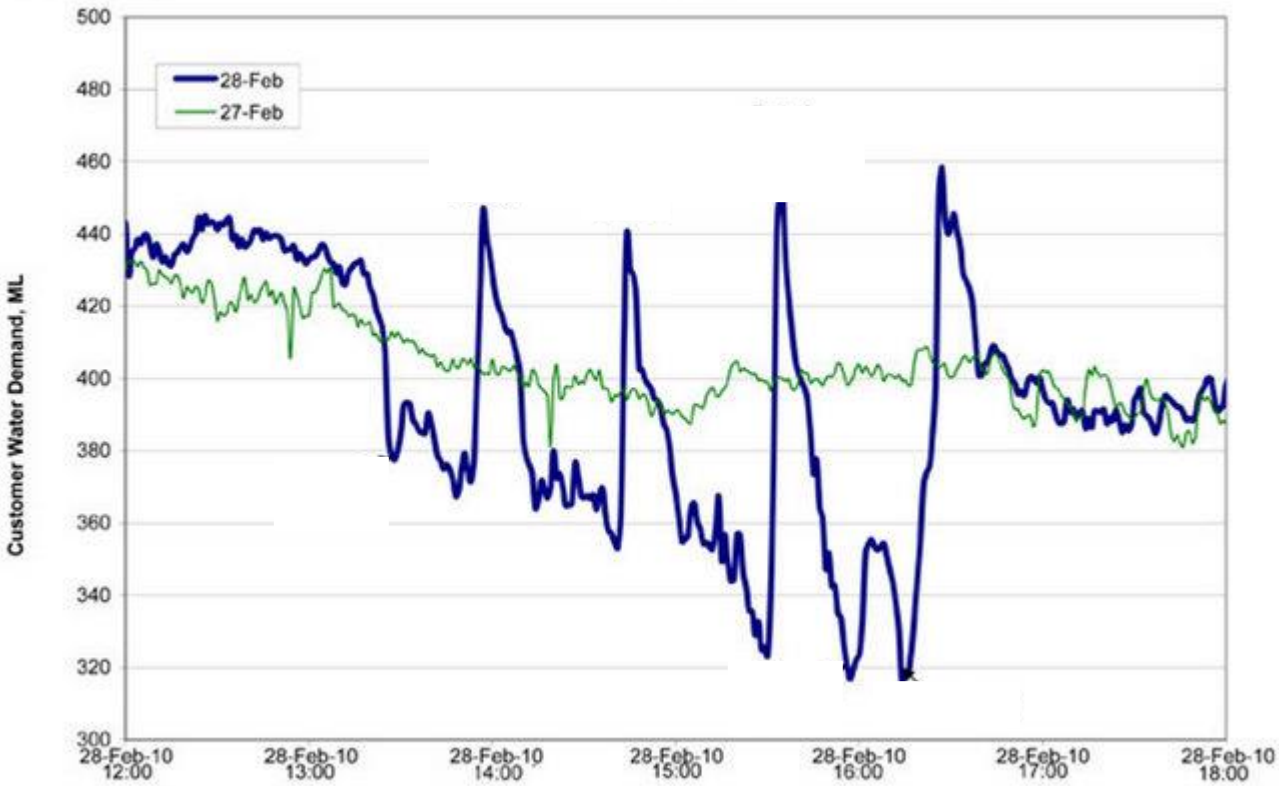
- A *time series* (ts) is a set of observations taken sequentially in time
- A ts can be represented as a set

  $\{x_t : t = 1,2,3,\ldots,n\} = \{x_1,x_2,x_3,\ldots,x_n\}$
- For example,

  {10,31,27,NA,53,15}
- Univariate (e.g. total # of fish caught) or multivariate (e.g. # of each species caught)

Example of a time series of water usage typical versus during hockey championship game

Water Usage



Time of Day

How do we describe a time series?

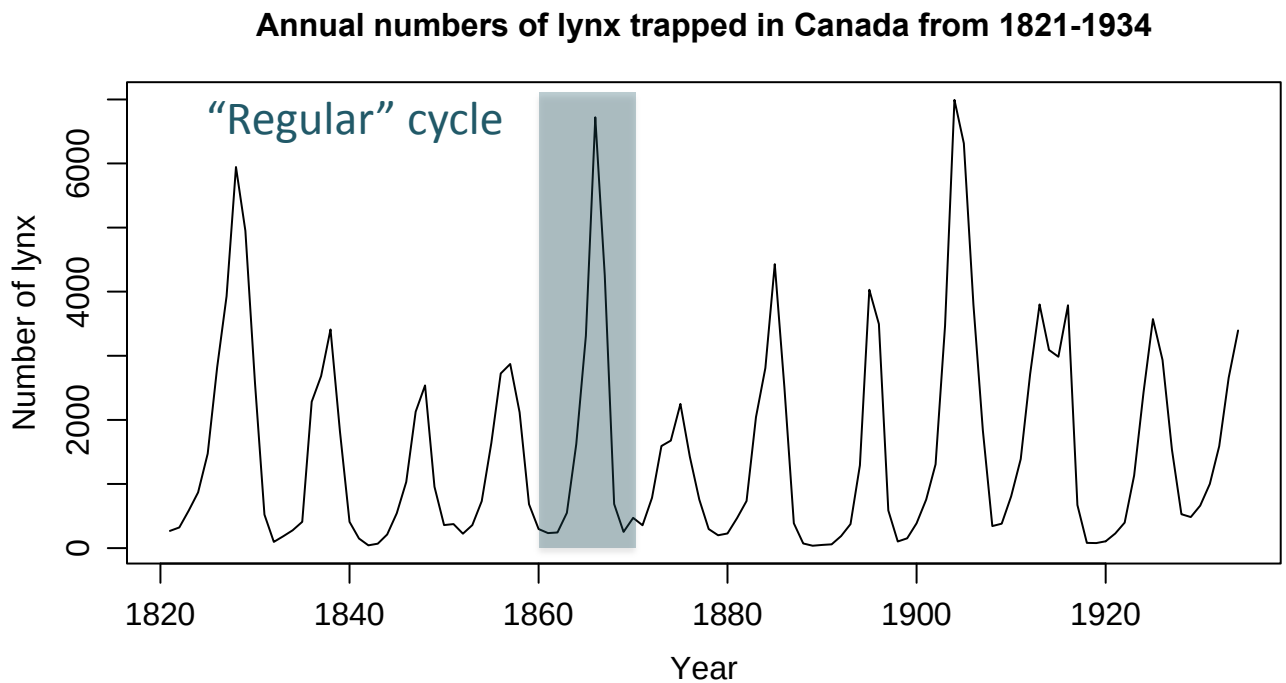
$$observation_t = \text{trend} + \text{cycle} + e_t$$

$$e_t = \text{a time series also}$$

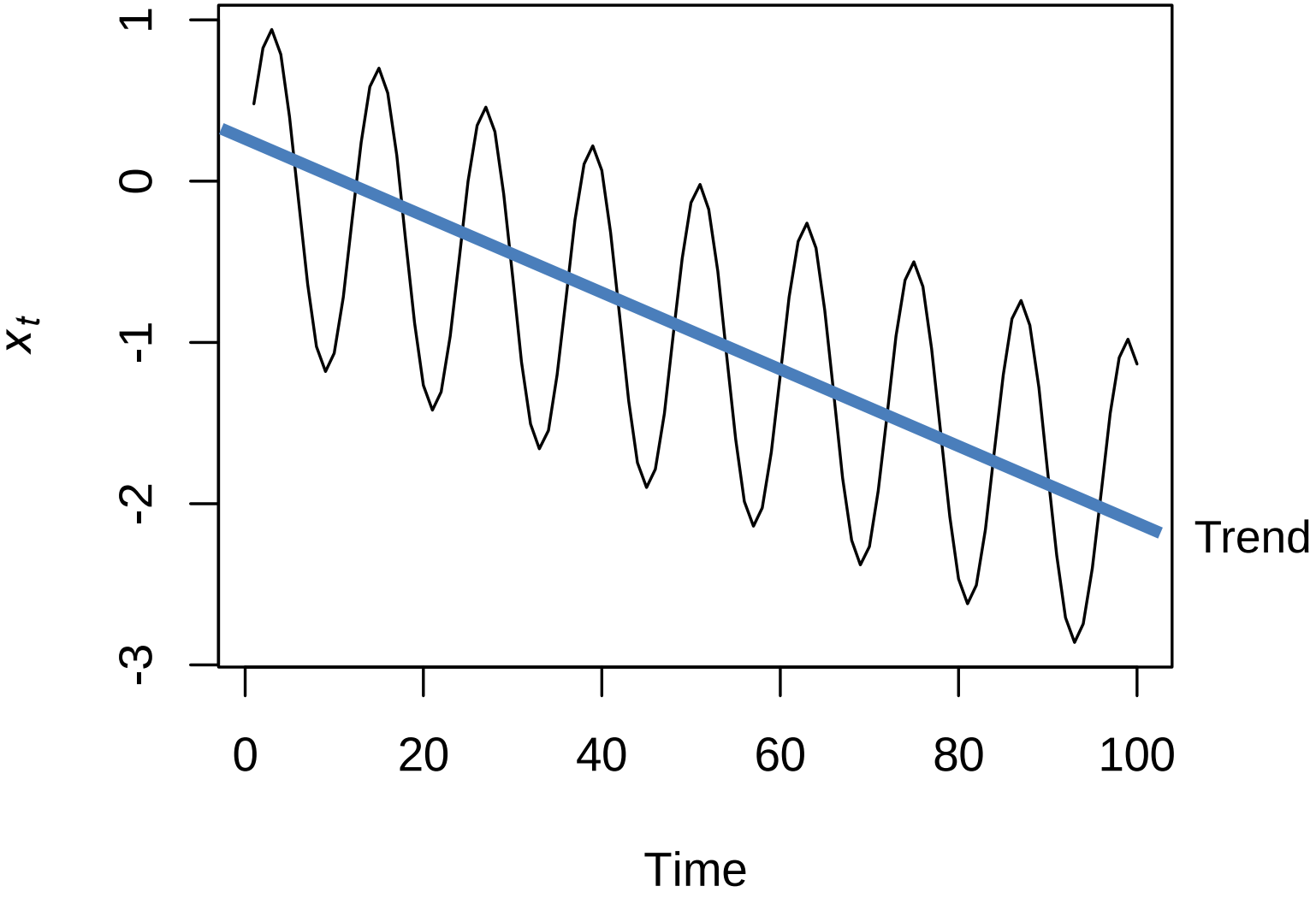
Often the objective is to estimate or describe the trend and cycle in a time series, but to do this we need to describe/model the e_t.

Other times the objective is to model the e_t since we are trying to understand what drives the year-to-year (month-to-month) variation.

Cycles or seasonality in a time series



Trend in a time series



Stationarity of time series

- *Stationarity* describes a particular statistical properties of a time series.
- In general, a time series is said to be stationary if there is
  1) no systematic change in mean or variance,
  2) **no systematic trend up or down, and**
  3) **no periodic variations or seasonality**

We typically remove the trend and cycles and treat e_t as stationary.

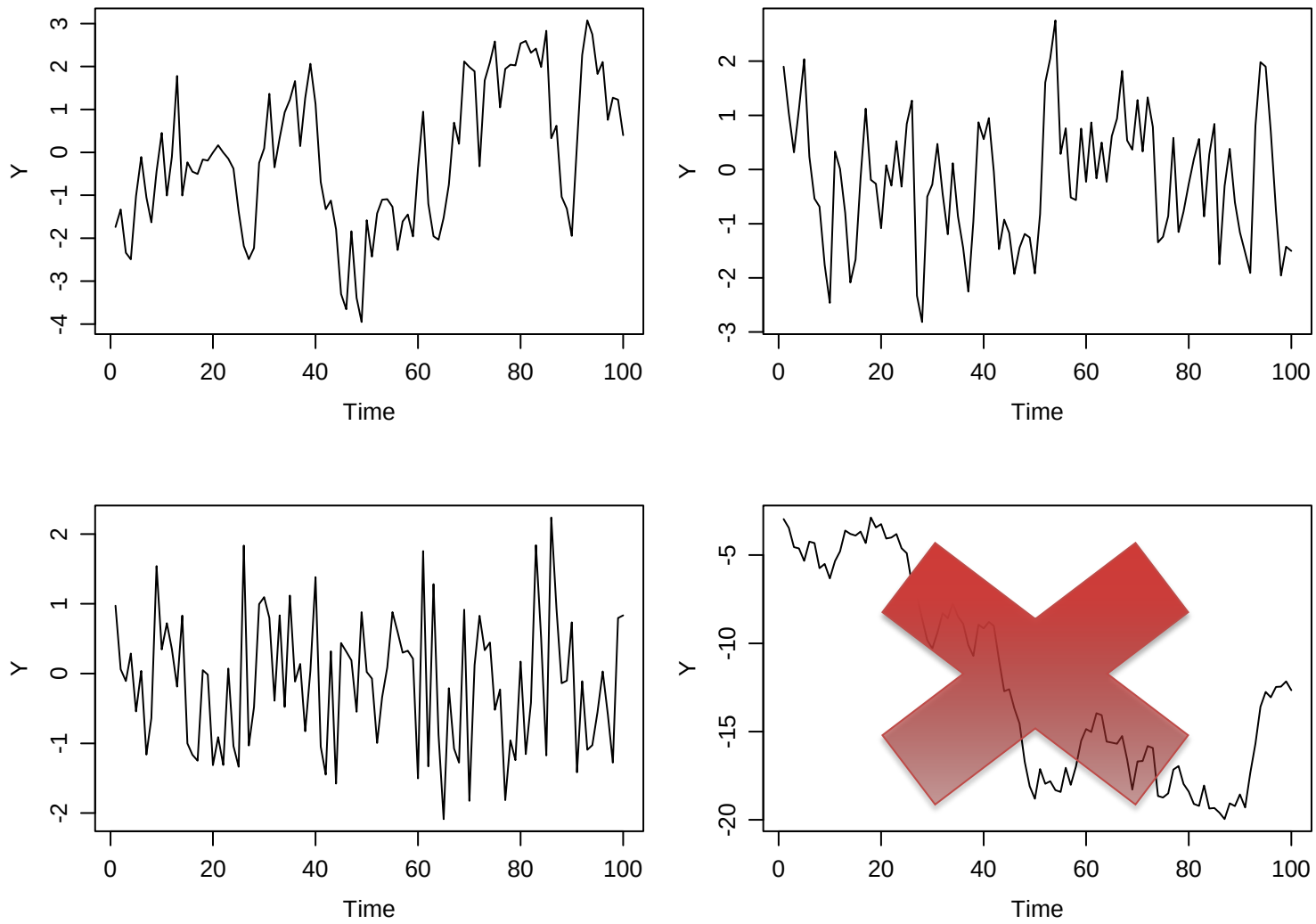
Describing a time series: classical decomposition

- *Classical decomposition* of an observed time series is a fundamental approach in time series analysis
- The idea is to decompose a time series $\{x_t\}$ into a trend, a seasonal component, and a remainder (e_t)

$$observation_t = \text{trend} + \text{cycle} + e_t$$

$$e_t = \text{a time series also}$$

Which of these are stationary?

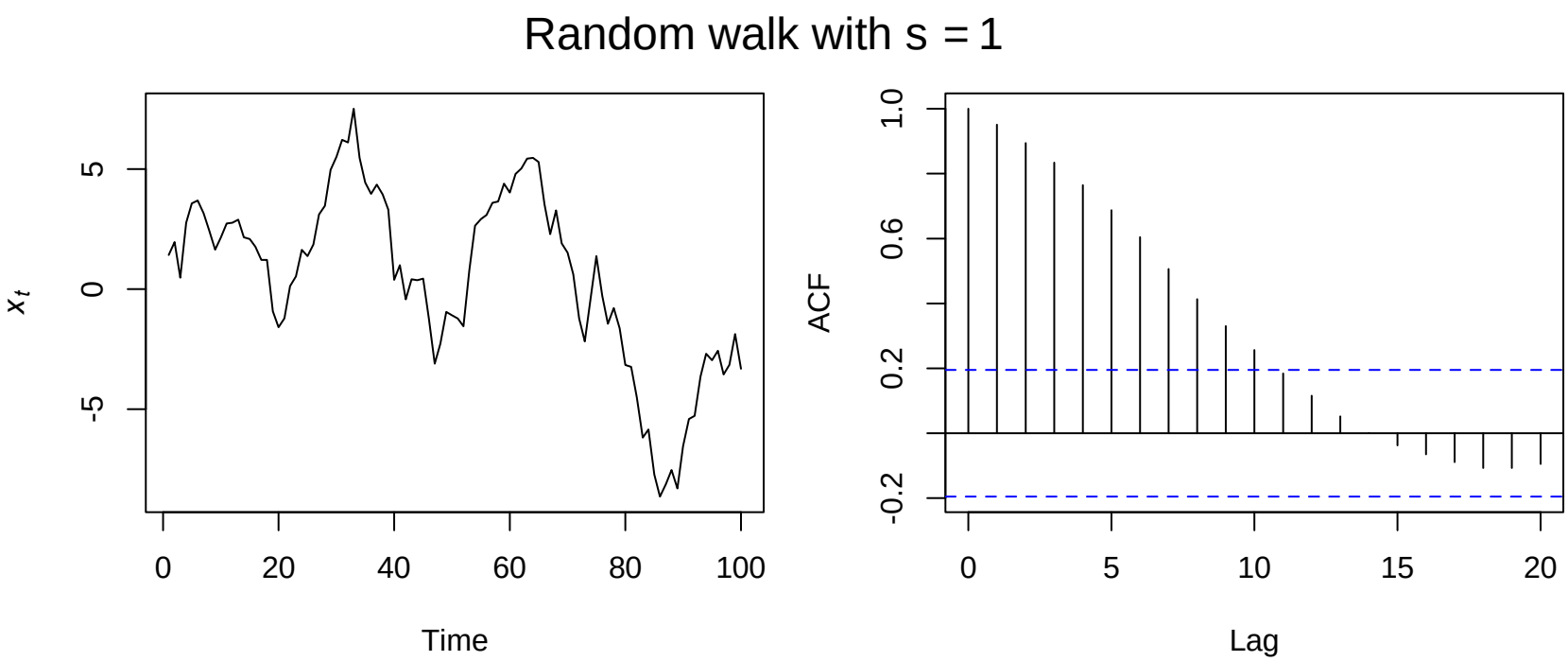


Classic time-series models

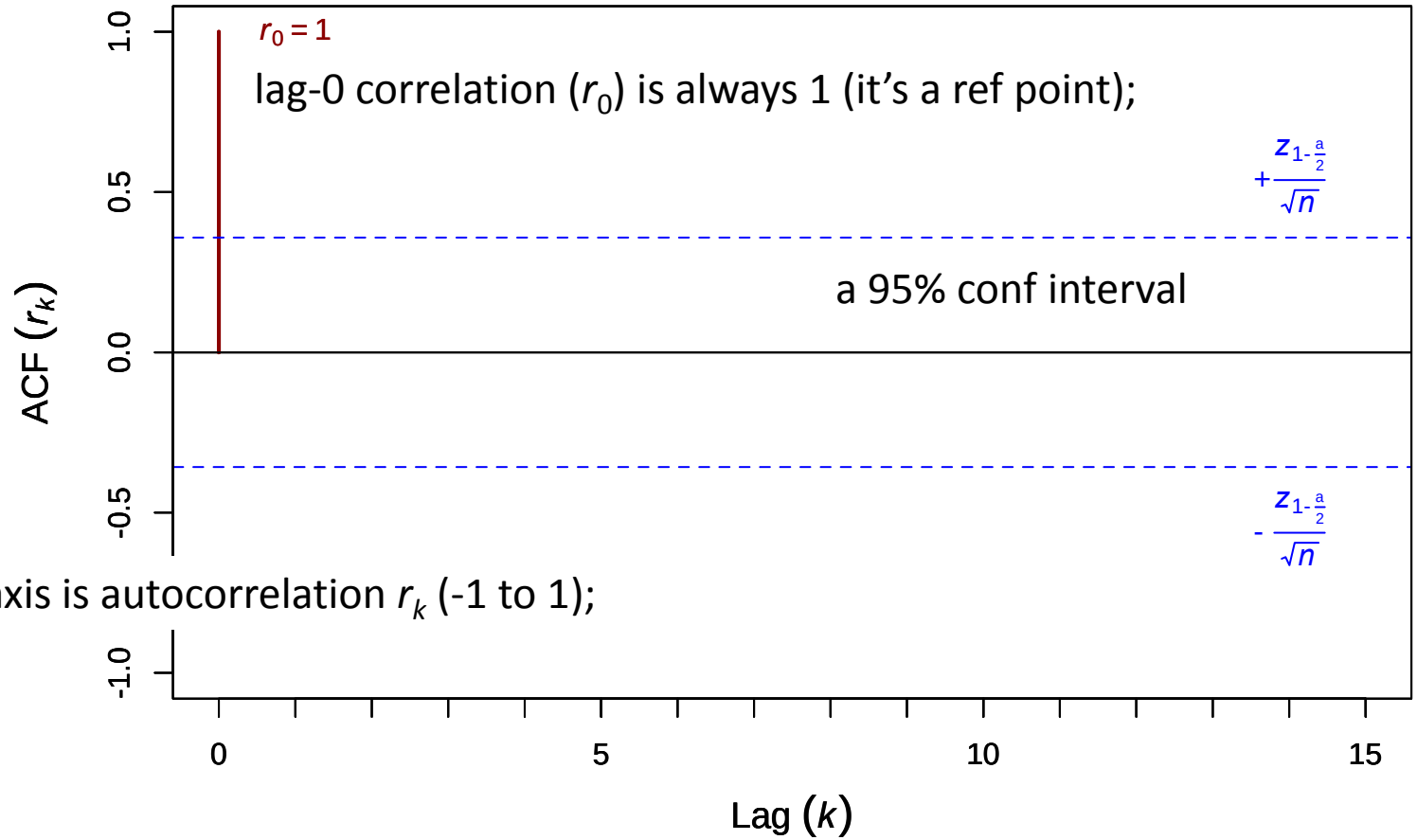
- A *time series model* for $\{x_t\}$ is a specification of the joint distributions of a sequence of random variables $\{X_t\}$ of which $\{x_t\}$ is thought to be a realization.
    - Time-series models – these describe the e_t
        - White noise
        - Autoregressive (AR) models
        - Moving average (MA) models
        - ARMA models
    - Random walks – an important type of non-stationary ts

Autocorrelation function (ACF): a powerful way to summarize a ts

- ACF measures the correlation of a time series against a time-shifted version of itself (& hence “auto”)

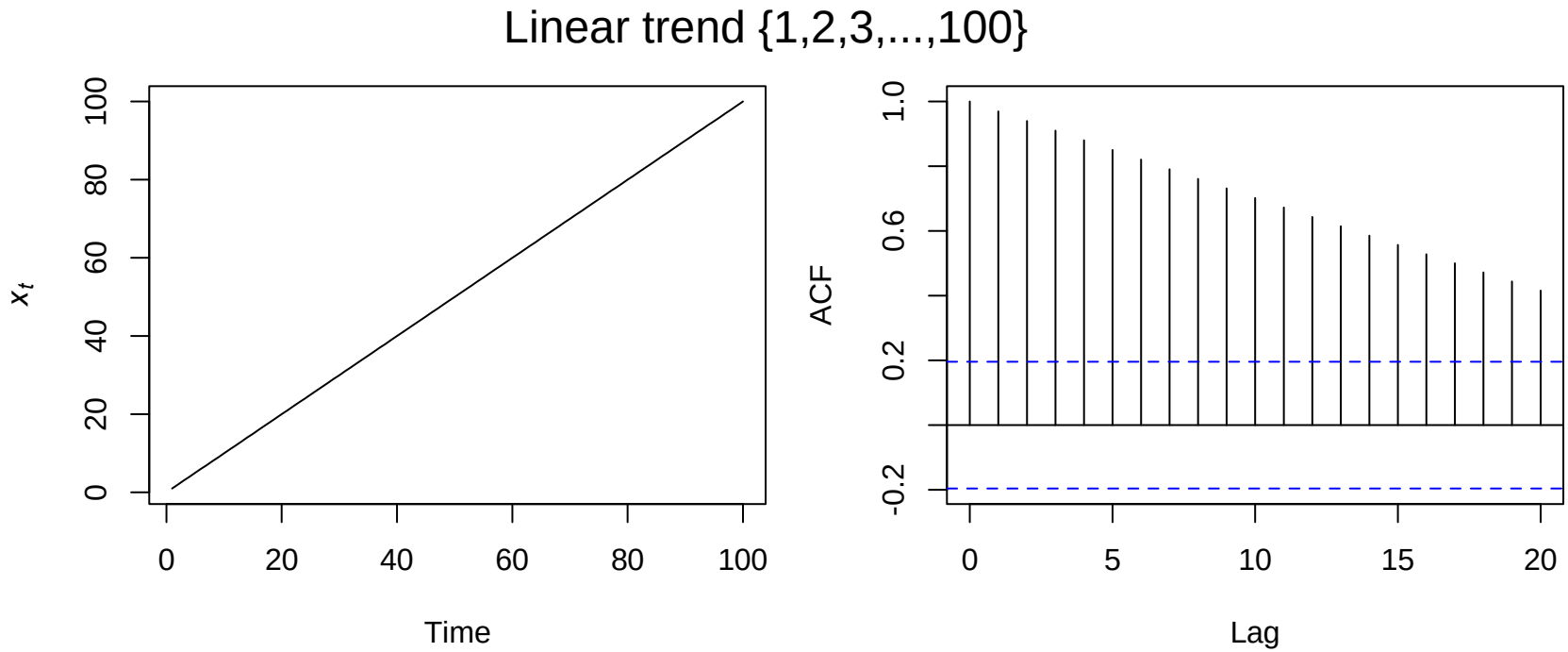


The correlogram



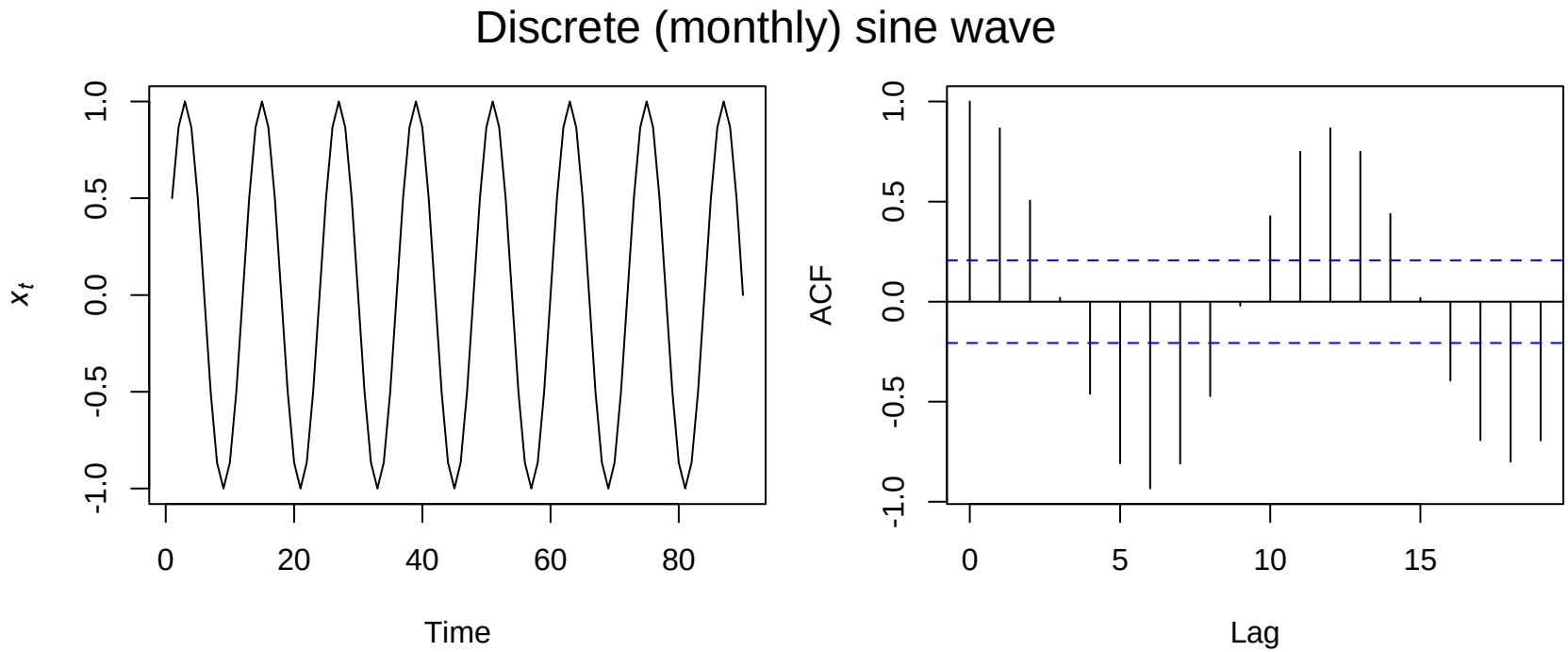
y-axis is autocorrelation r_k (-1 to 1);

Correlogram for deterministic trend



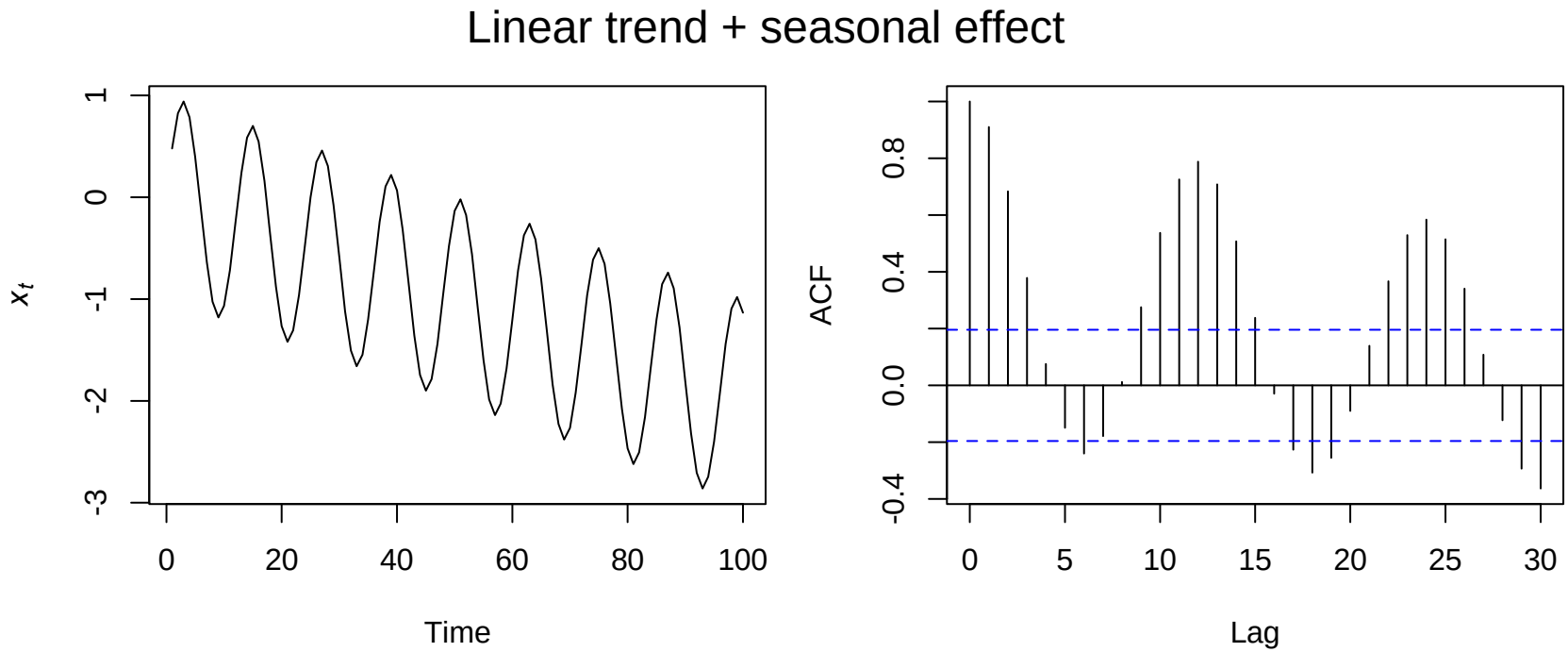
Non-stationary

Correlogram for sine wave



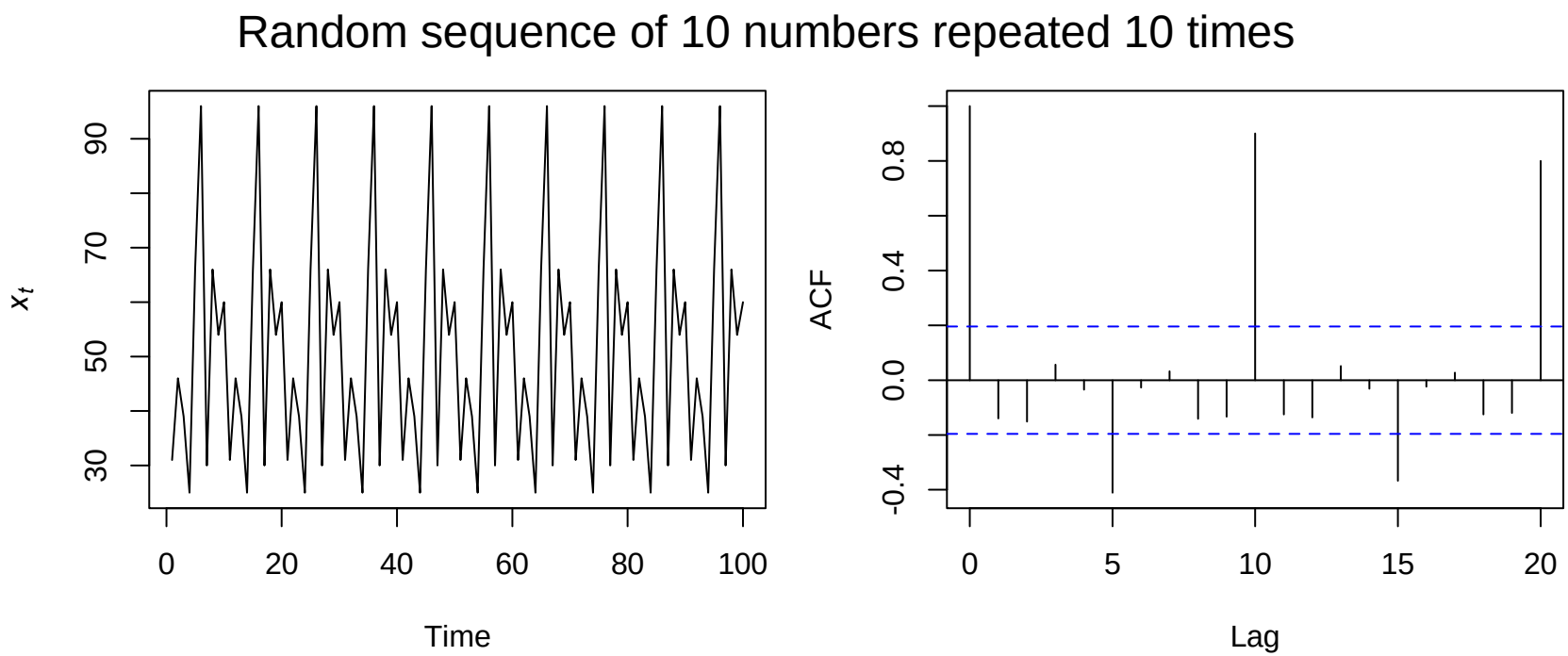
Non-stationary

Correlogram for trend + season



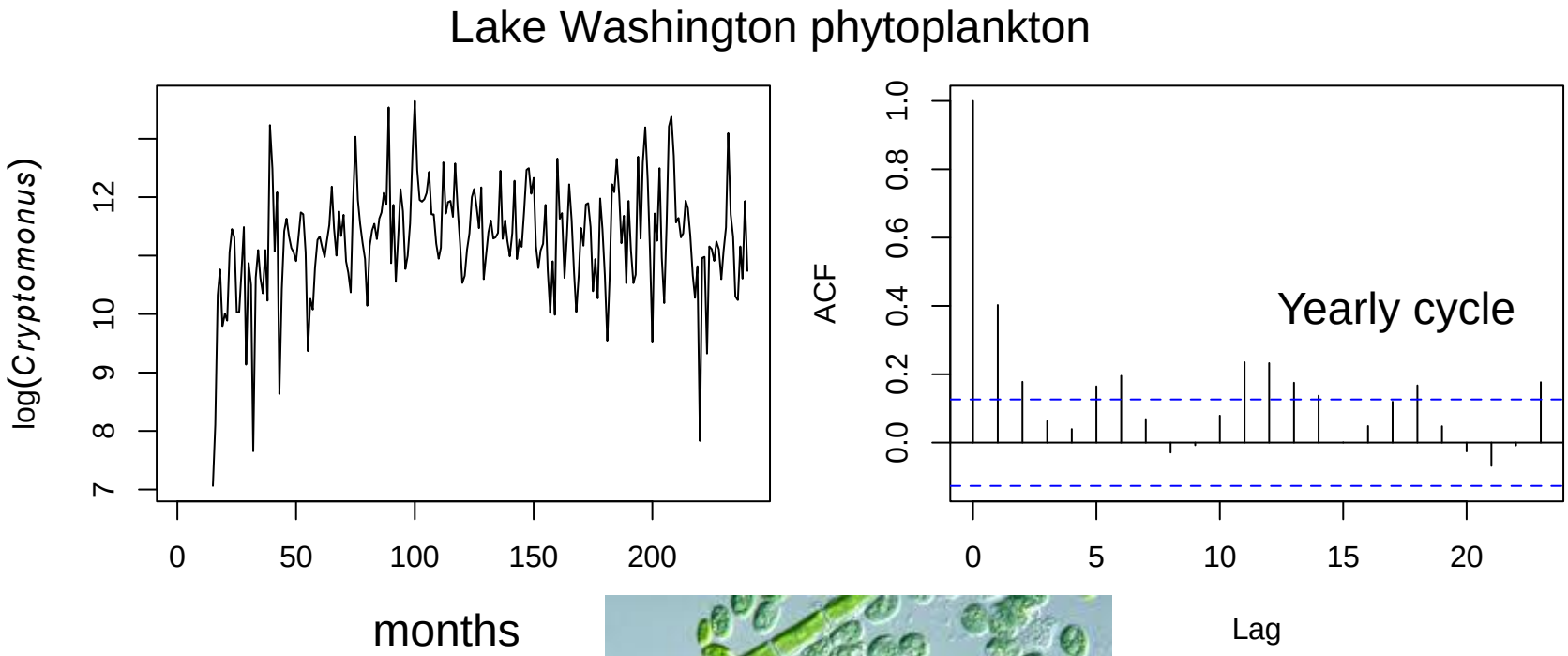
Non-stationary

Correlogram for random sequence



Non-stationary

Correlogram for real data



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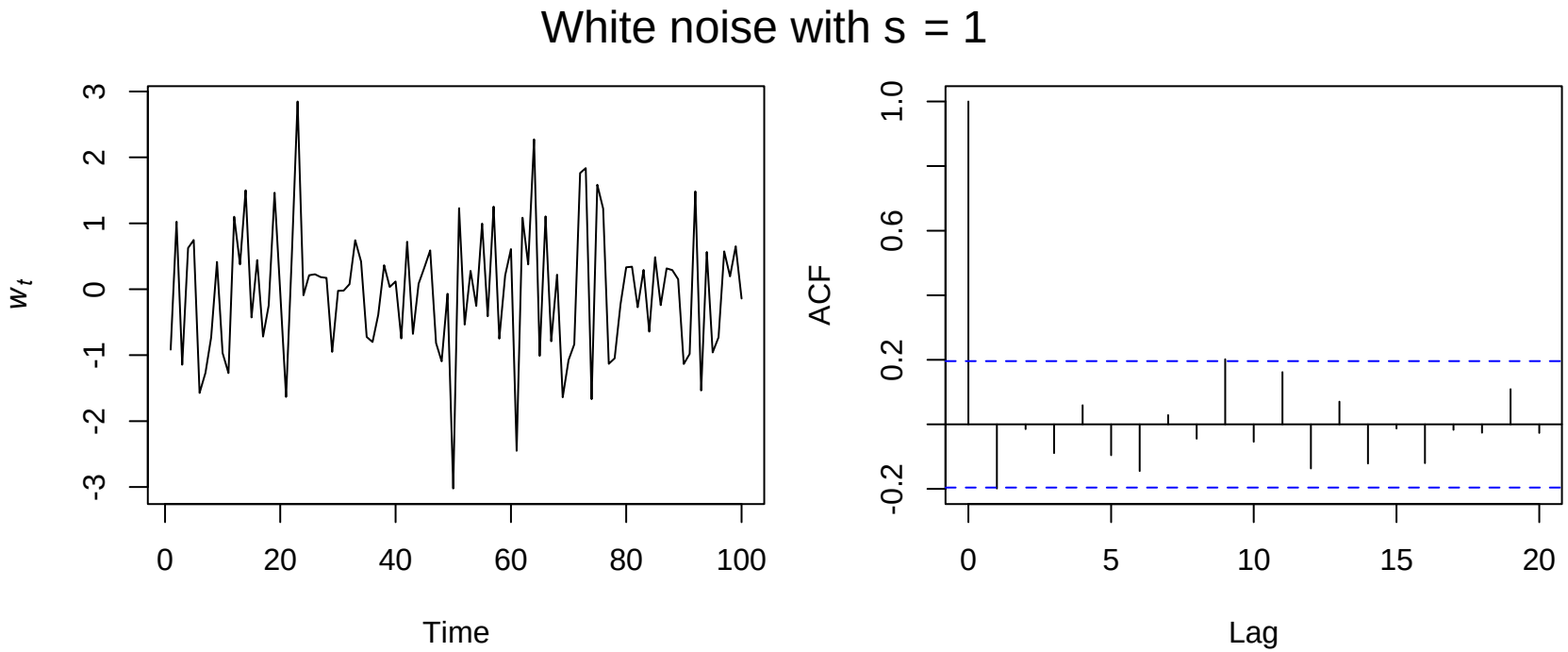
White noise

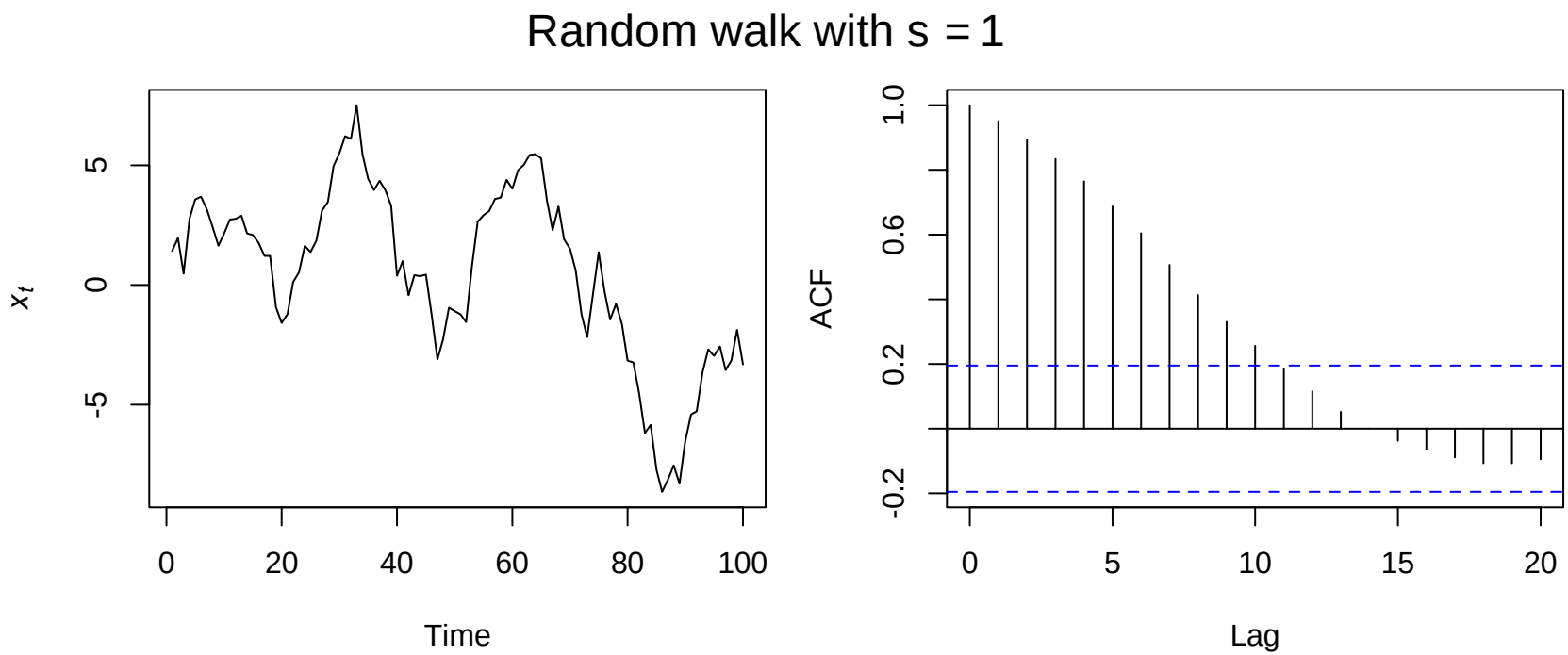


white noise x_t are

1) *independent*, and

2) *identically distributed* with a mean of zero

Random walk (RW)



A time series $\{x_t : t = 1,2,3,\ldots,n\}$ is a *random walk* if

1) $x_t = x_{t-1} + w_t$, and

2) w_t is white noise

Random walks are NOT stationary!

Autoregressive (AR) models

- An *autoregressive* model of order *p*, or AR(*p*), is defined as

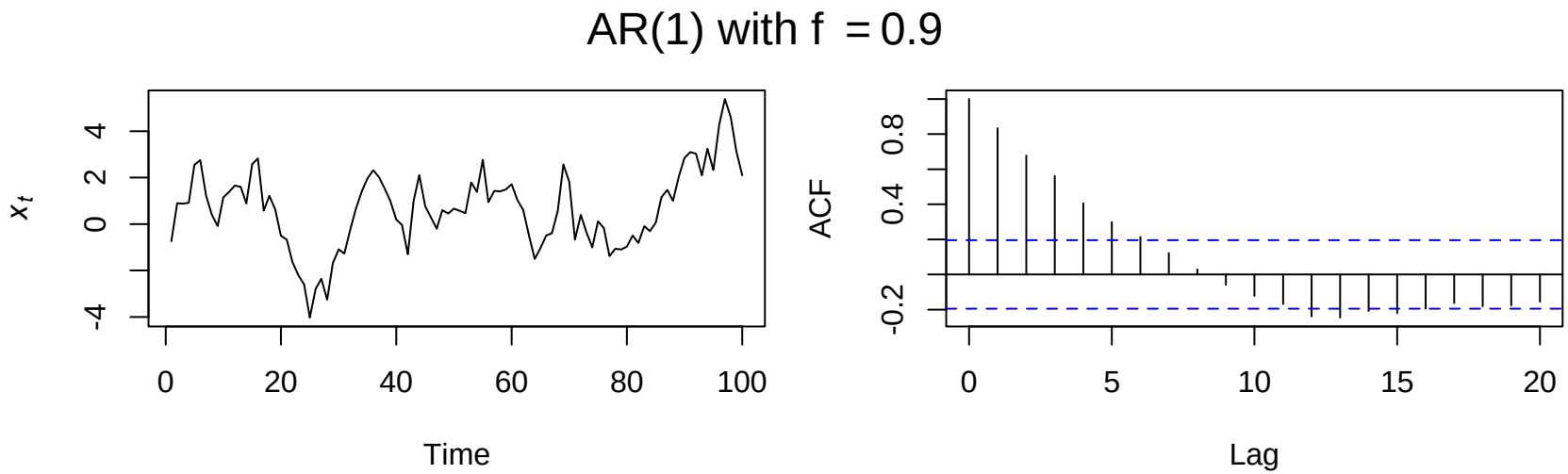
$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p} + w_t$$

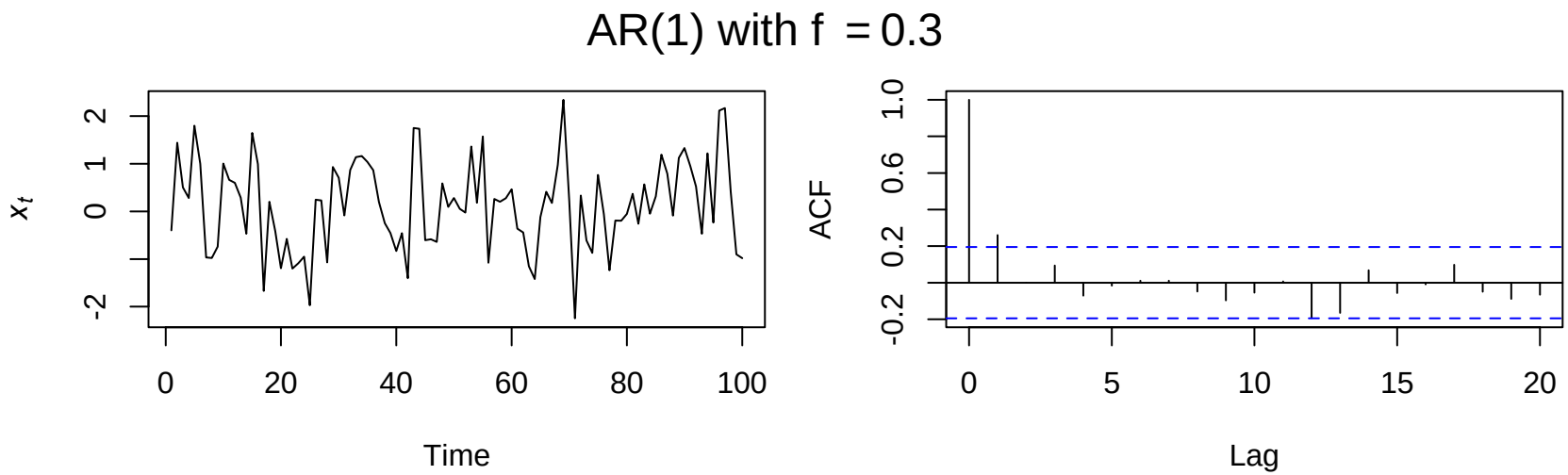
where we assume

1) w_t is WN, and
2) $\phi_p \neq 0$ for order-*p* process

- *Note*: RW model is special case of AR(1) with $\phi_1 = 1$

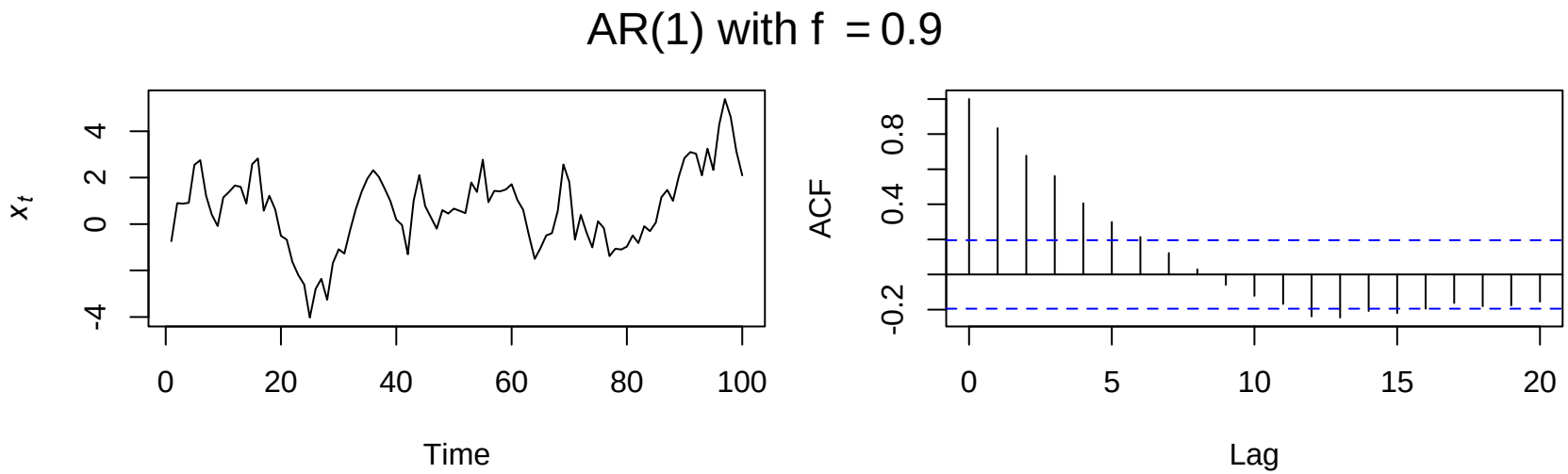
Examples of AR(1) processes

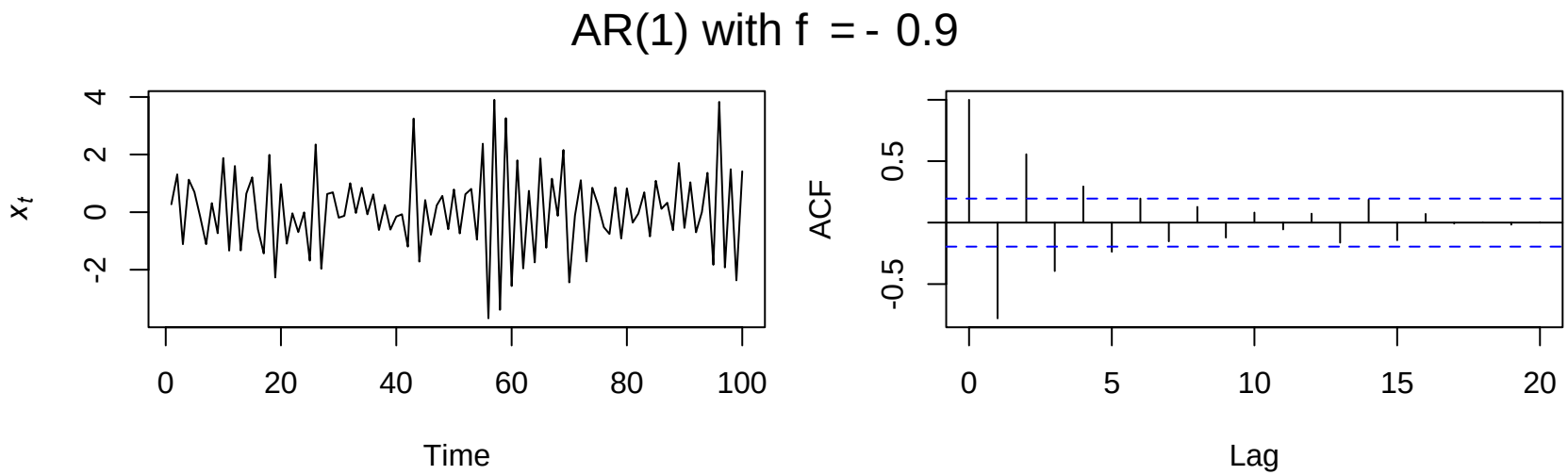




AR(p) can be stationary!

Examples of AR(1) processes





Partial autocorrelation function

- The partial *autocorrelation function* (PACF) measures the linear correlation of a series x_t and x_{t+k} with the linear dependence of $\{x_{t-1}, x_{t-2}, \ldots, x_{t-(k-1)}\}$ removed
- It is defined as

$$\phi_{kk} = \begin{cases} \mathrm{Cor}(x_1, x_0) = \rho(1) & \text{if } k = 1 \\ \mathrm{Cor}\left(x_k - x_k^{k-1}, x_0 - x_0^{k-1}\right) & \text{if } k \geq 2 \end{cases} \qquad -1 \leq \phi_{kk} \leq 1$$

$$x_k^{k-1} = \phi_1 x_{k-1} + \phi_2 x_{k-2} + \cdots + \phi_{k-1} x_1$$

$$x_0^{k-1} = \phi_1 x_1 + \phi_2 x_2 + \cdots + \phi_{k-1} x_{k-1}$$

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$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p} + w_t$$

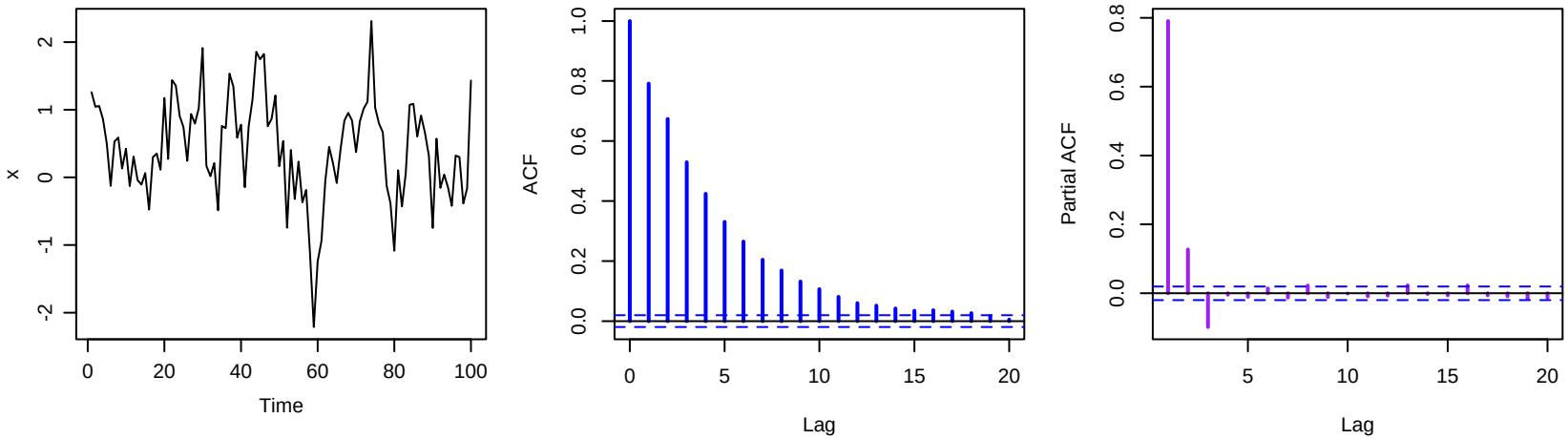
where we assume

1) w_t is WN, and
2) $\phi_p \neq 0$ for order-*p* process

- *Note*: RW model is special case of AR(1) with $\phi_1 = 1$

ACF & PACF for AR(3) processes

AR(3) with $f_1 = 0.7$, $f_2 = 0.2$, $f_3 = -0.1$



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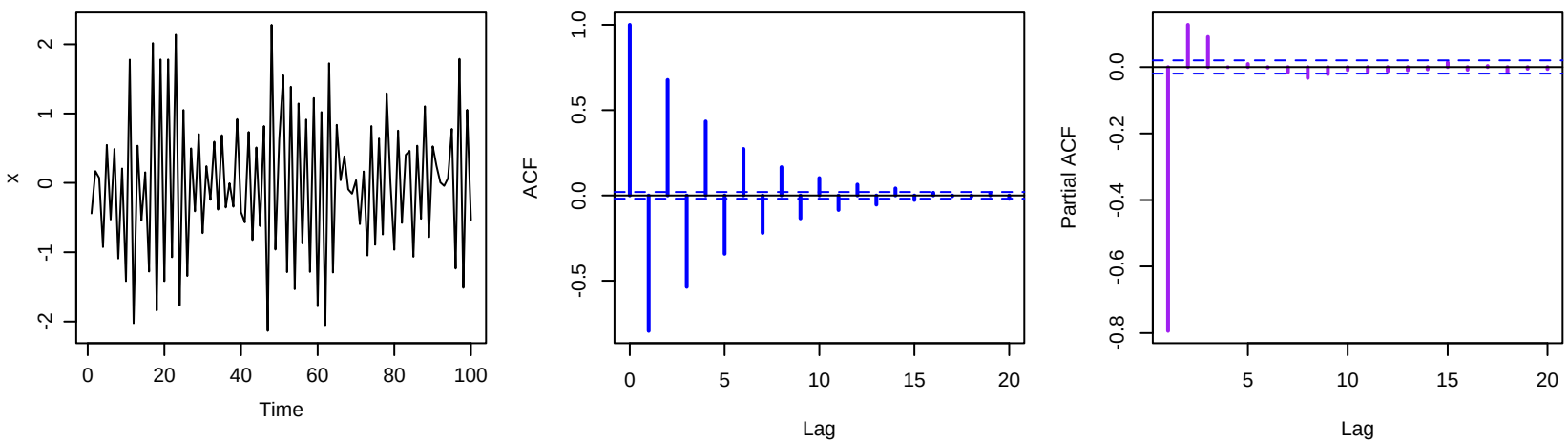
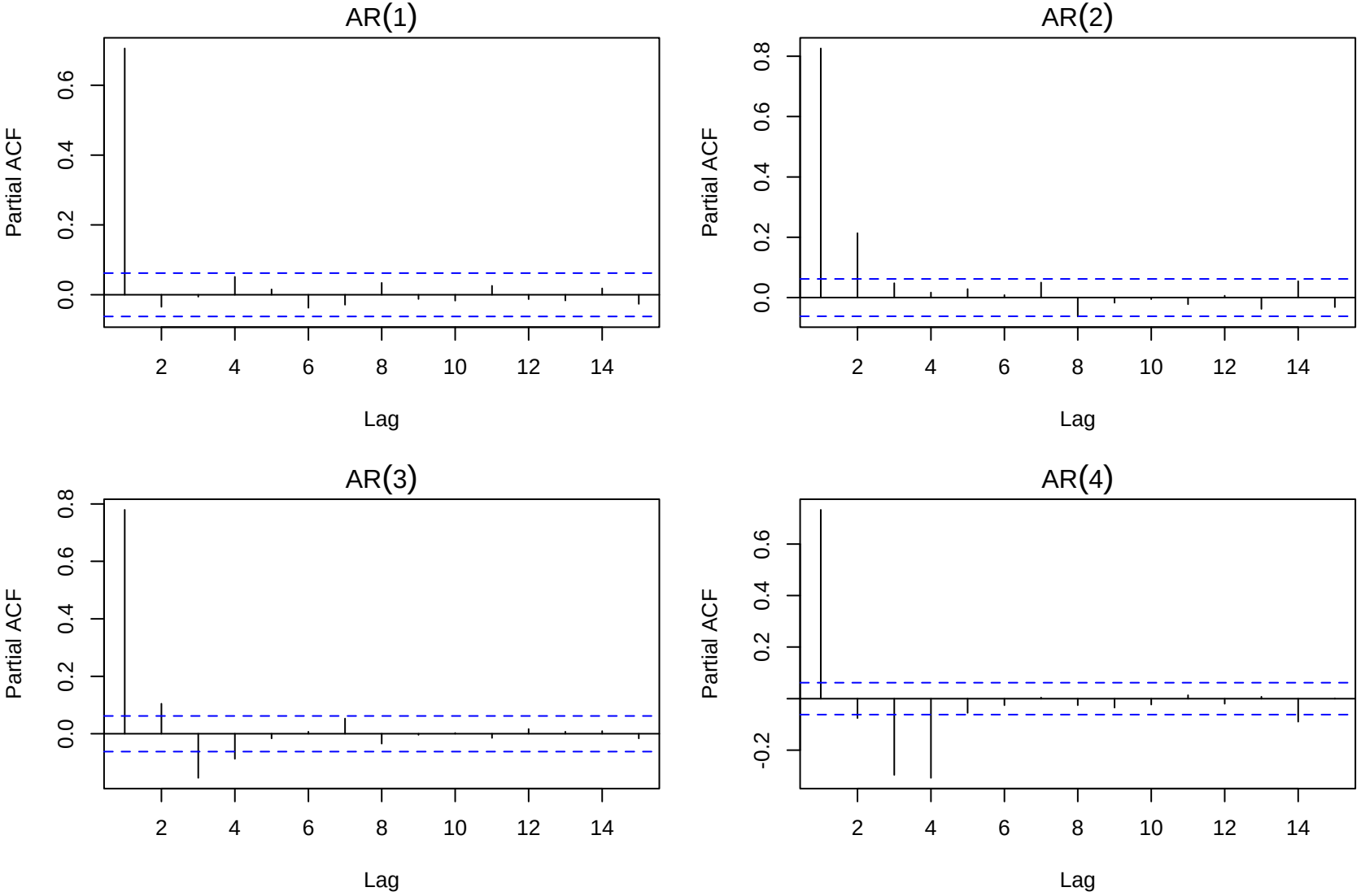


PACF for AR(p) processes



Moving average (MA) models

- A *moving average* model of order q, or MA(q), is defined as

$$x_t = w_t + \theta_1 w_{t-1} + \ldots + \theta_q w_{t-q}$$

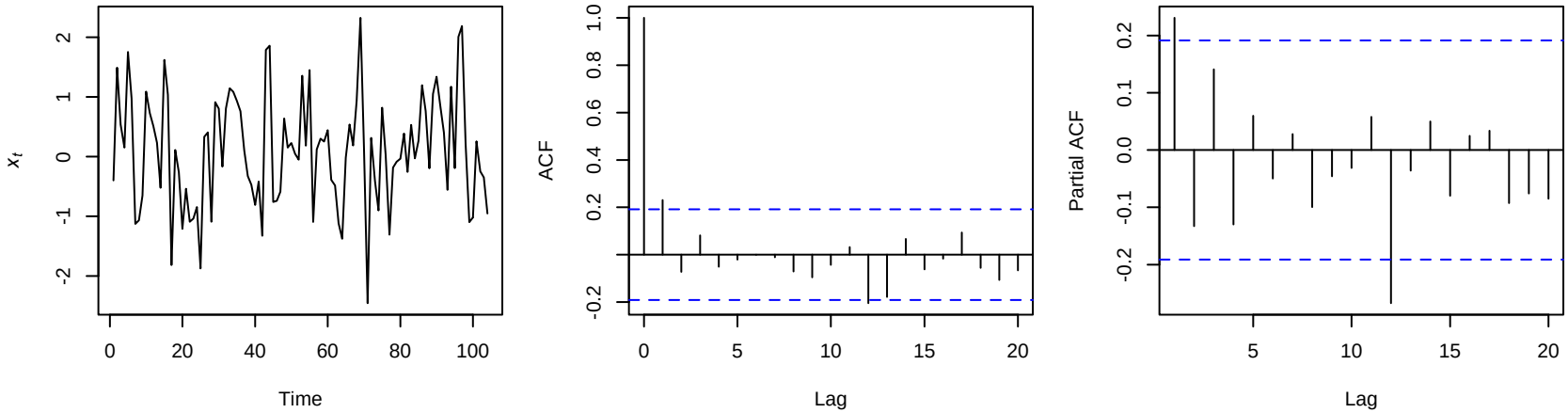
  where w_t is WN (with 0 mean)

- It is simply the current error term plus a weighted sum of the q most recent error terms

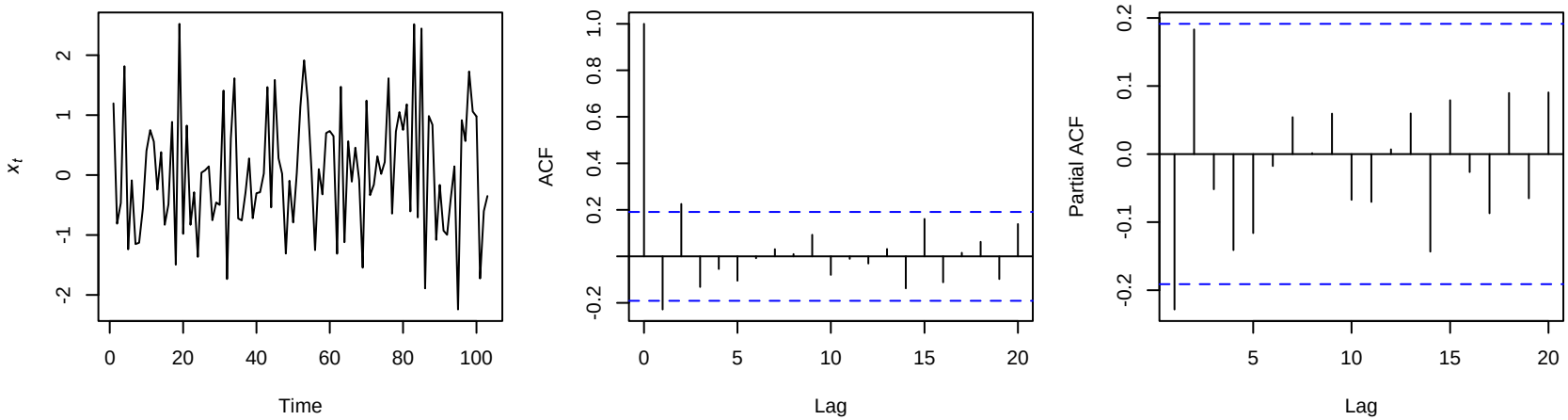
- Because MA processes are finite sums of stationary WN processes, they are themselves stationary

Examples of MA(q) processes

MA(1) with q = 0.3



MA(2) with q_1 = -0.3, q_2 = 0.3



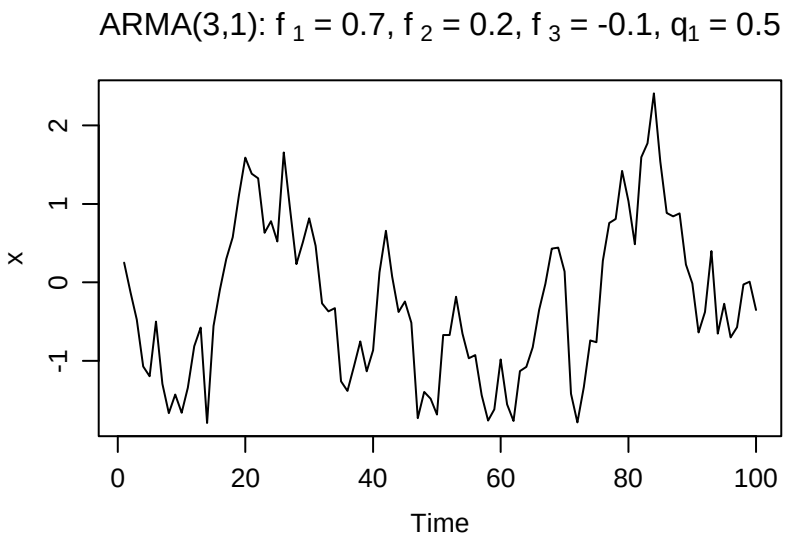
Autoregressive moving average models

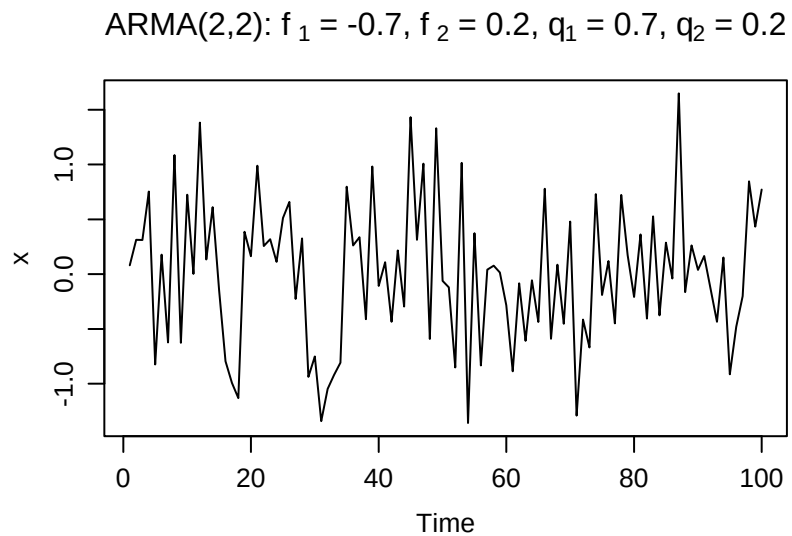
- A time series is *autoregressive moving average*, or ARMA(p,q), if it is stationary and

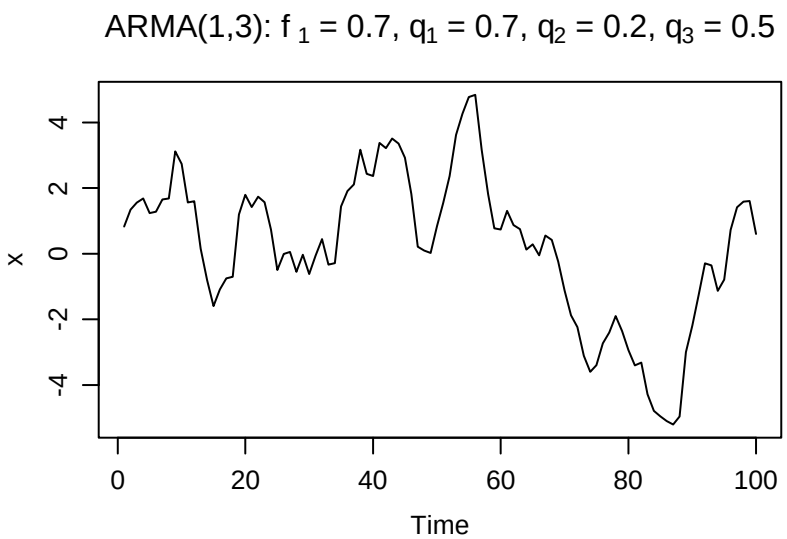
$$x_t = \phi_1 x_{t-1} + \cdots + \phi_p x_{t-p} + w_t + \theta_1 w_{t-1} + \cdots + \theta_q w_{t-q}$$

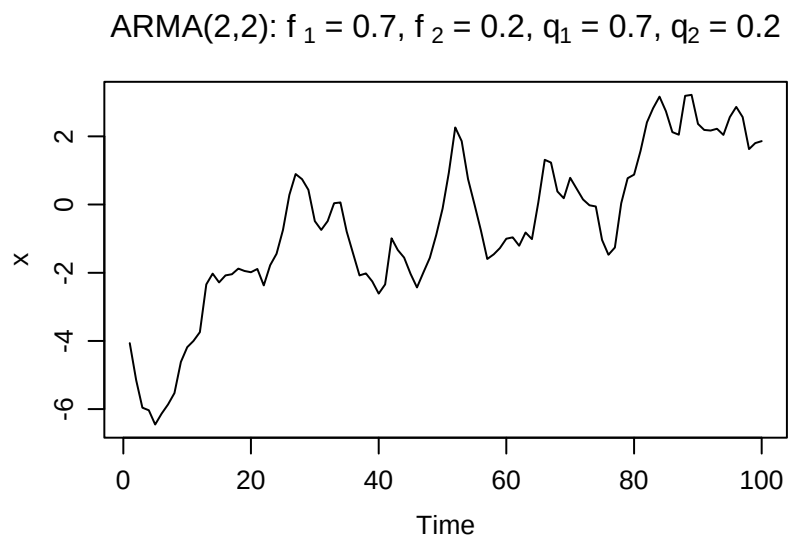
- Combines both AR(p) and MA(q)

Examples of ARMA(*p*,*q*) processes

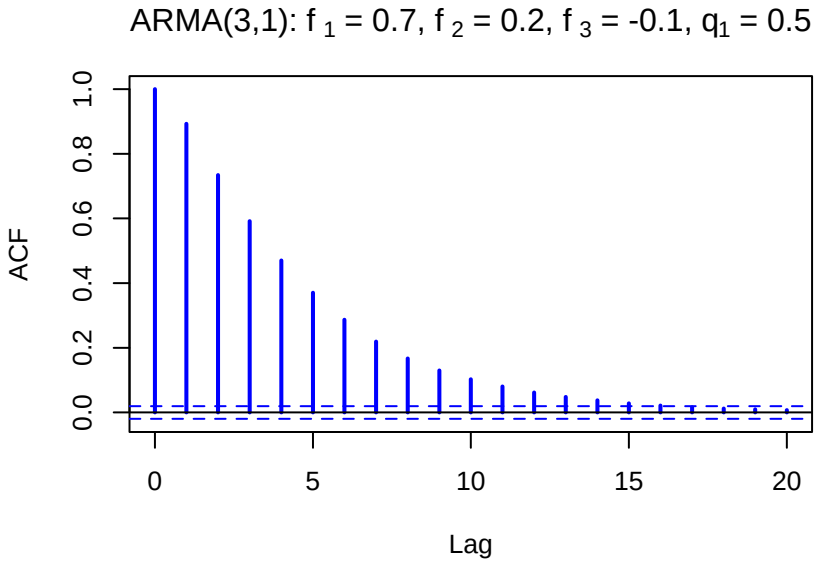


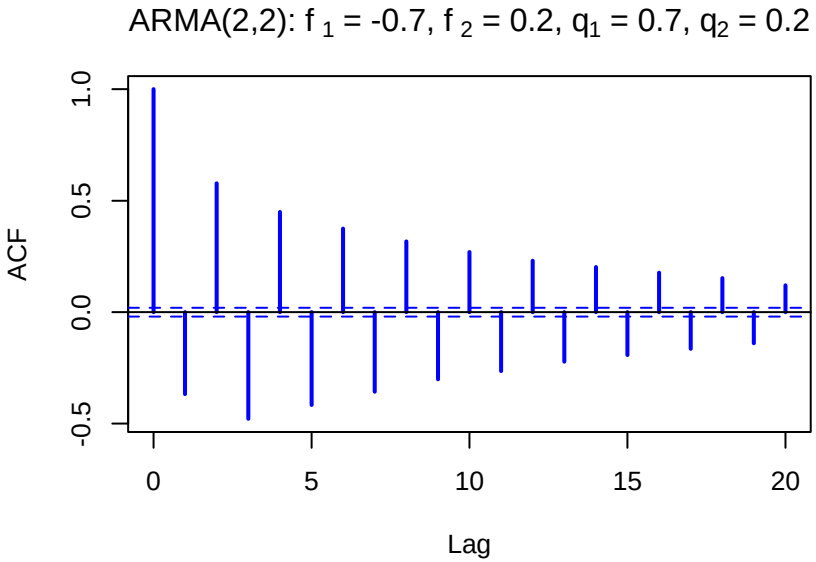


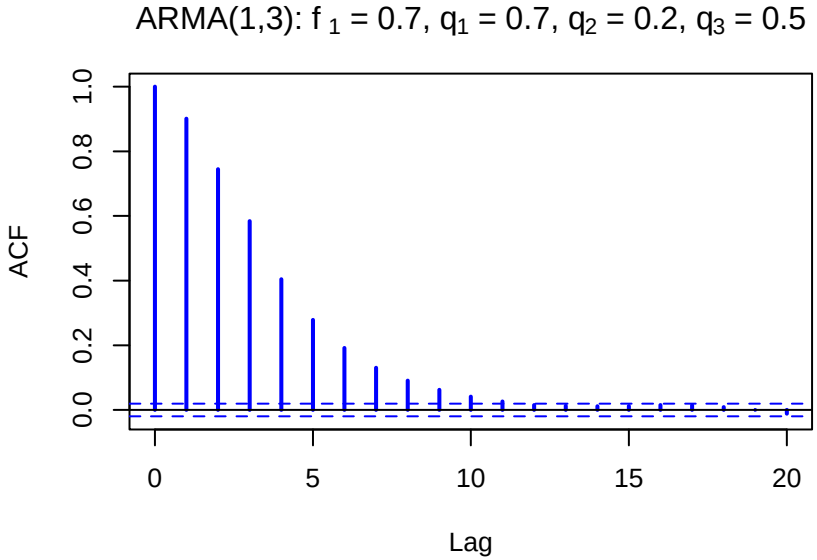


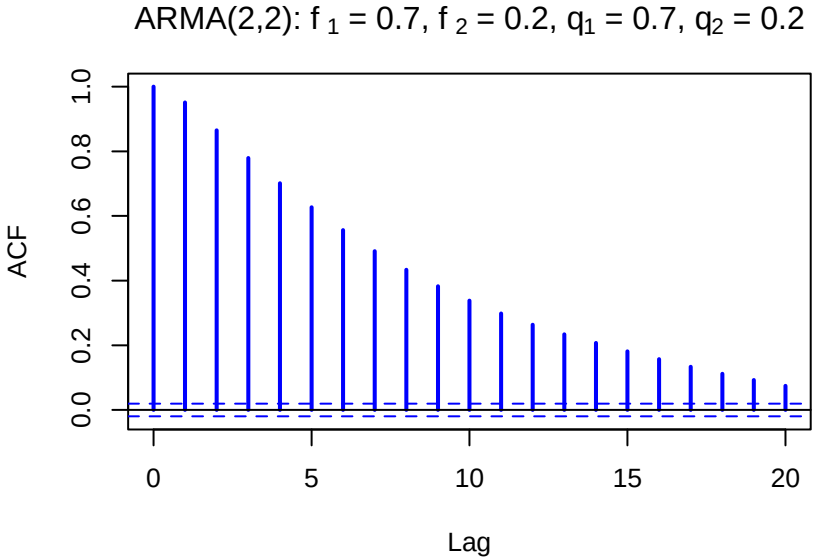


ACF for ARMA(p,q) processes

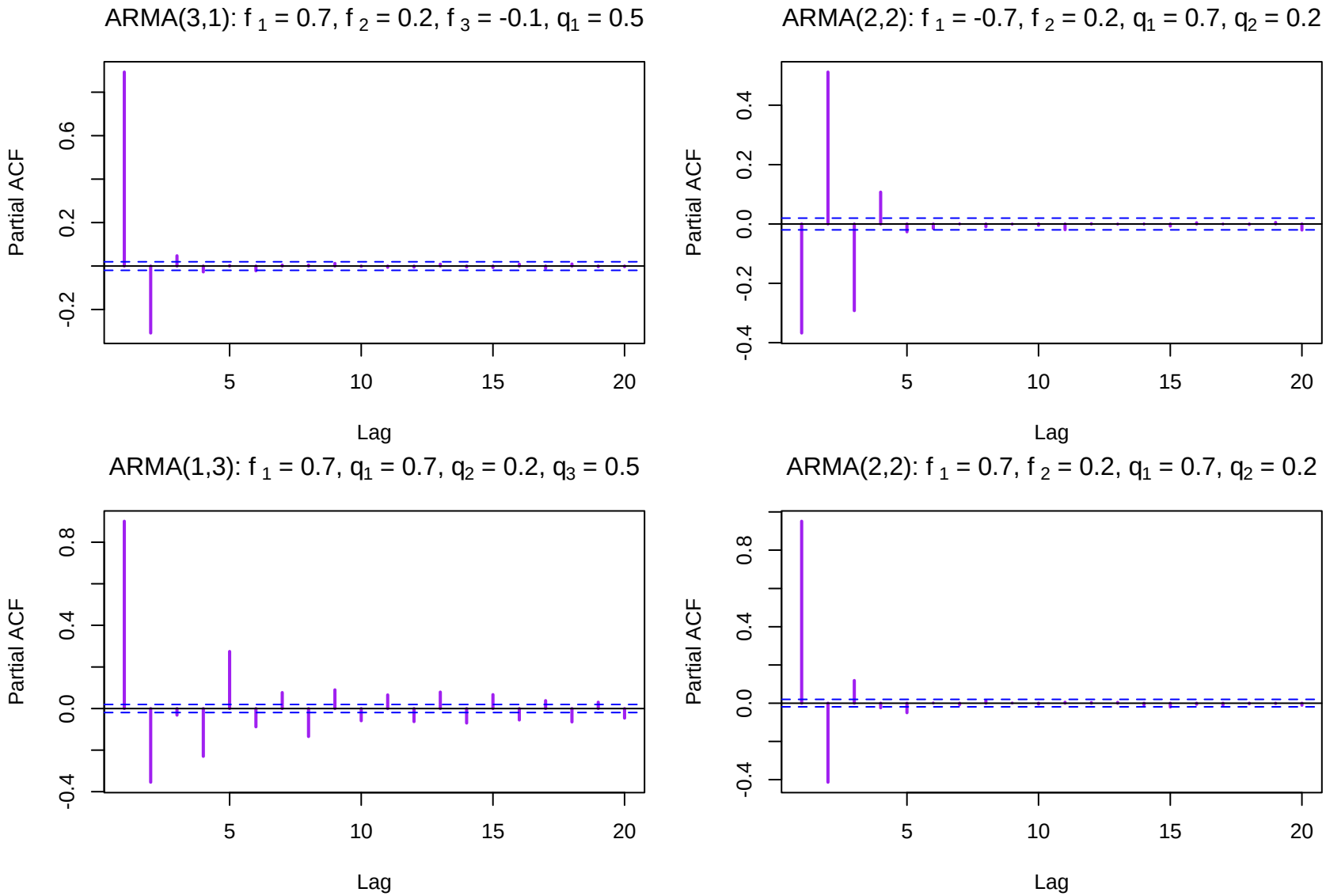








PACF for ARMA(p,q) processes



Difference to remove trend/season

- Differencing is a very simple means for removing a trend or seasonal effect
- The 1st-difference removes a linear trend, a 2nd-difference would remove a quadratic trend, etc.
- For seasonal data, using a 1st-difference with *lag = period* removes both trend & seasonal effects
- Pro: no parameters to estimate
- Con: no estimate of stationary process

Using ACF & PACF for model ID

	ACF	PACF
AR(p)	Tails off	Cuts off after lag-p
MA(q)	Cuts off after lag-q	Tails off
ARMA(p,q)	Tails off (after lag [q-p])	Tails off (after lag [p-q])

Topics for this lab

- `ts` class in R
- Plotting ts objects
- Understand covariance & correlation
- Examine some simple ts models
- Use `diff()` for trend/season removal
- Examine properties via `acf()` & `pacf()`
- Examine AR(p) models
- Examine MA(q) models
- ARMA(p,q) models via 'arima.sim()'